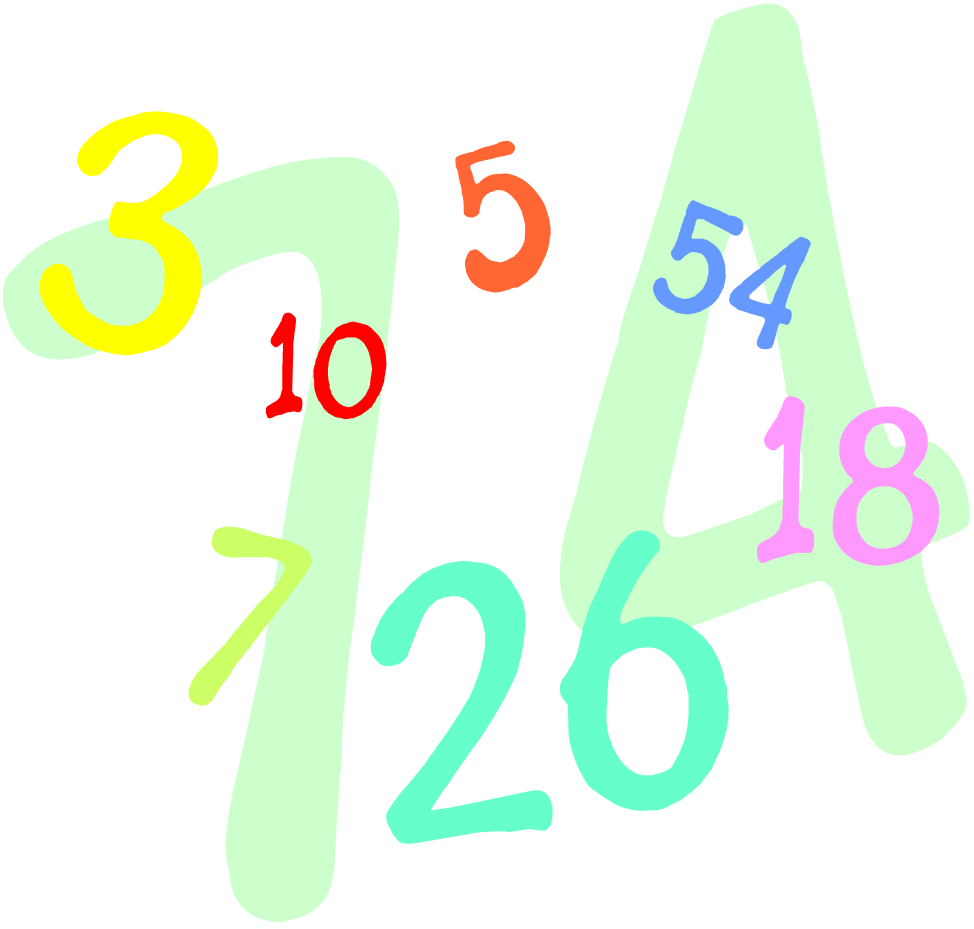
Burford Primary School

Supporting your child with Mathematics

"They don't learn maths in school like I did – what is going on?"

Calculation



Calculation

- ✓ This booklet has been designed for parents of children at Burford Primary School.
- ✓ Its aim is to explain and show how the four operations of calculation (addition, subtraction, multiplication and division) are taught in school.
- ✓ Useful vocabulary can be found at the back of the booklet.

The aim of teaching calculation is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

Introduction

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. (Oxfordshire Maths Team – Calculation Policy)

It is important that children constantly practise their mental maths knowledge in order to support their written work. Please practise number bonds and times tables facts at home.

Please note the leaflet explains when certain skills are taught in school. However, no two children are the same and there are those who will access skills earlier than their peers and those who will access them later. Please take the suggested year group teaching as guidance only.

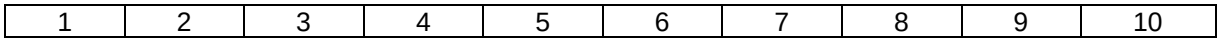
Addition

To add successfully, children need to be able to:

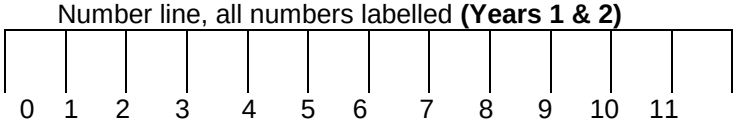
- recall all addition pairs to 9 + 9 and complements in 10, such as □ + 3 =10;
- add mentally a series of one-digit numbers, (such as 5 + 8 + 4);
- use addition facts, (such as 6 + 7), and their knowledge of place value to add multiples of 10 (such as 60 + 70) or of 100, (such as 600 + 700)
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Progression in the use of number lines: The number line is an incredibly powerful tool that can be used for all four number operations. At Burford we teach the children to use these from as early as the Foundation Stage. Children who can picture a number line in their head have a powerful tool to use for mental calculations. The following diagrams show the progression of number lines used:

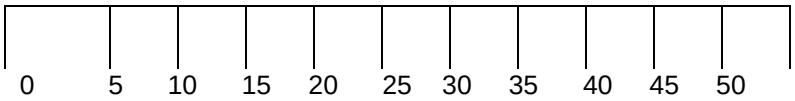
Number track **(Foundation Stage)**



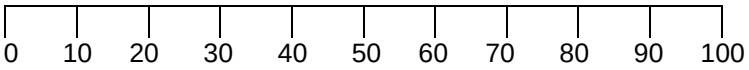
Number line, all numbers labelled **(Years 1 & 2)**



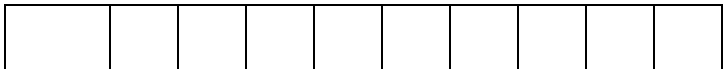
Number line, 5s and 10s labelled (**Years 1 & 2)**



Number lines, 10s labelled **(Years 1 & 2)**



Number lines, marked but unlabelled **(Years 1 & 2)**



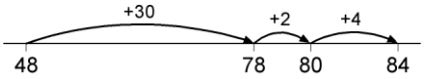
Empty number line (**Years 3 & 4)**

Once children have a sound understanding of how a number line works, they can begin to use it to add and subtract numbers at around Year 3, as shown in Stage 1 below, although some children begin to do this earlier and others need support beyond Year 3.

Stage 1: The Empty Number Line (Year 3/4): Children are taught to record steps in addition on a number line. The steps often bridge through a multiple of 10. That is to say the 7 has been partitioned into 5 and 2. The 2 is added first to reach the next multiple of 10 then the remaining 5 has been added.

8 + 7 = 15

48 + 36 = 84



or:

Stage 2: Partitioning (Year 3/4):

- This method records mental methods, using partitioning into tens and units separately: tens, then the units are added to form partial sums and then the partial sums are added. For example:

47 + 76
47 + 70 = 117 then
117+ 6 = 123

- Partitioning both numbers into tens and units, mirrors the column method where units are placed under units and tens under tens. This also links to mental methods. For example:

47 + 76
40 + 70 = 110 then
7 + 6 = 13 then
110 + 13 = 123

Partitioned numbers are then written under one another, for example:

$$\begin{array}{rll} & 47= & 40 + 7 \\ + & 76= & 70 + 6 \\ \hline & & 110+13 \quad =123 \end{array}$$

Stage 3: Expanded Method in Columns (Year 3/4):

This method shows the addition of the tens to the tens and the units to the units separately. To find the partial sums, initially the tens, not the units, are added first, following mental methods. It is important that when adding the tens that the child recognises they are calculating forty plus seventy equals one hundred, but stressing the link to the related fact four plus seven equals eleven. The total of the partial sums can be found by adding them together. For example:

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$$

The adding of the units first gives the same answer as adding the tens first. As children gain confidence, they are encouraged to add the units first.

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$$

Stage 4: Compact Column Method (Year 5/6):

In this method, recording is reduced further. ‘Carry’ digits are recorded below the line, using the words ‘carry ten’ or ‘carry one hundred’, not ‘carry one’. For example:

$$\begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ {\scriptstyle 1\,1} \end{array} \qquad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ {\scriptstyle 1\,1} \end{array}$$

Subtraction This is known as the inverse or opposite of addition and the two are often taught together.

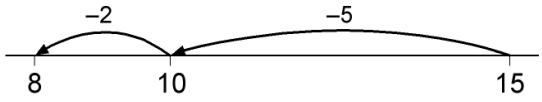
To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20; to be able to count backwards;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact, 16 – 7, and their knowledge of place value;
- have a good number knowledge for example the closeness of two numbers could mean that it is more efficient to count on rather than back;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

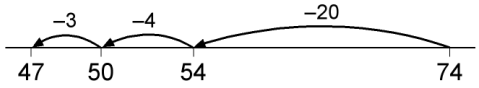
From foundation stage subtraction or take away is taught practically and through songs and rhymes. During key stage one practical apparatus are used such as cubes and other counters. From year three the recording and calculating becomes more formal:

Stage 1: Empty Number Line (Year 3/4): Children are taught to record steps on an empty number line. The steps often bridge through a multiple of 10: they count back to the next multiple of ten and then take away the remaining part of the number. In the example below, 7 has been split into 5 and 2 so that the 5 can be taken away to reach the next multiple of ten, then the remaining two, to reach 8:

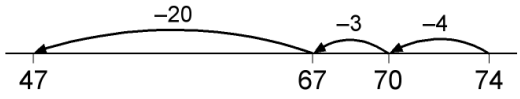
15 – 7 = 8

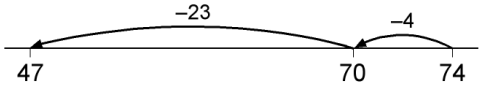


74 – 27 = 47 worked by counting back:



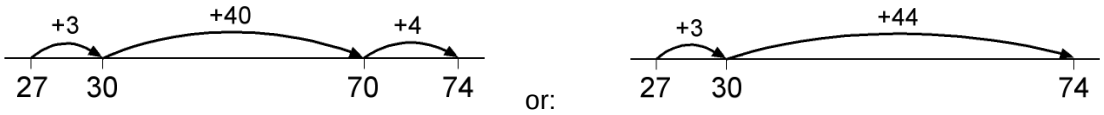
The steps may be recorded in a different order: or combined:





The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74, in steps totalling 47 (shopkeepers method):

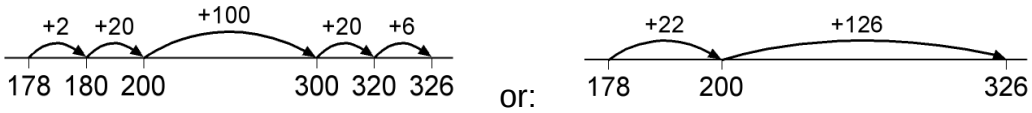
74 – 27 =



• With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.

With three-digit numbers the number of steps can again be reduced, enabling children to work out answers to calculations such as 326 – 178 first in small steps and then more compact by using knowledge of complements to 100. For example:

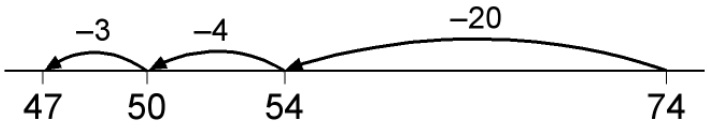
326 – 178 =



Stage 2: Partitioning (Year 3/4): Children are also taught that subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For example:

74 – 27 involves partitioning the 27 into 20 and 7; subtracting 20 from 74 and then 7.

This use of partitioning is a useful step towards the most commonly used column method.



Stage 3: Expanded layout, leading to column method: The next step, to teaching subtraction, is to partition the numbers and take like from like. For example 563 – 241 = can be done as follows:

Example 1, without any adjustment (Year 3/4): 563 – 241:

	500	60	3
-	200	40	1
	300	20	2

Start by subtracting the units, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying ‘sixty take away forty’, not six take away four.

Example 2, adjustment from the tens to the units (Year 3/4):

563 - 246

		50	13	
	500	~~60~~	~~3~~	
-	200	40	6	
	300	10	7	= 317

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and units components of the number; how there is a "snag" with the units and the need to exchange a ten. To release units, 60 + 3 can be partitioned into 50 + 13. The subtraction of the units becomes '13 minus 6'and the subtraction of the tens becomes 50 minus 40.

Example 3, adjustment from the hundreds to the tens, or partitioning the hundreds (Year 5/6):

563 − 271:

	400	160		
	~~500~~	~~60~~	3	
-	200	70	1	
	200	90	2	= 292

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and units components of the number, how there is a "snag" with the tens and the need to exchange a hundred to release needed tens. 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70' and subtraction of the hundreds 400 minus 200.

Example 4, adjustment from the hundreds to the tens and the tens to units (Year 5/6):

563 – 278:

	400	150		
		~~50~~	13	
	~~500~~	~~60~~	~~3~~	
-	200	70	8	
	200	80	5	= 285

Here both the tens and the units to be subtracted are bigger than both the tens and the units you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example 5: Dealing with zeros when adjusting (Year 5/6):

503 − 278

		90		
	400	~~100~~	13	
	~~500~~	~~0~~	~~3~~	
-	200	70	8	
	200	20	5	= 225

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

Stage 4: compact method. Only children who are fully confident with the expanded method above will move onto this method of subtraction.

Example 1: no adjustment or decomposition needed:

563 − 241

	500 60 3	563
-	200 40 1	-241
	300 20 2	322

Start by subtracting the units, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying ‘sixty take away forty’, not ‘six take away four’

Example 2: adjustment from the tens to the units

563 − 246

		50	13			5 1
	500	~~60~~	~~3~~			5 ~~6~~ 3
-	200	40	6		-	2 4 6
	300	10	7	=317		3 1 7

Example 3: adjustment from the hundreds to the tens, or partitioning the hundreds

563 − 271

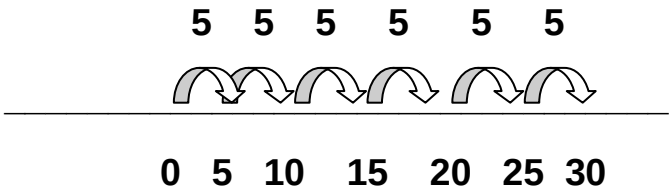
	400	160				41
	~~500~~	~~60~~	3		→	~~5~~63
-	200	70	1			271
	200	90	2	= 292		292

Multiplication To multiply successfully, children need to be able to recall all multiplication facts to 10 × 10;

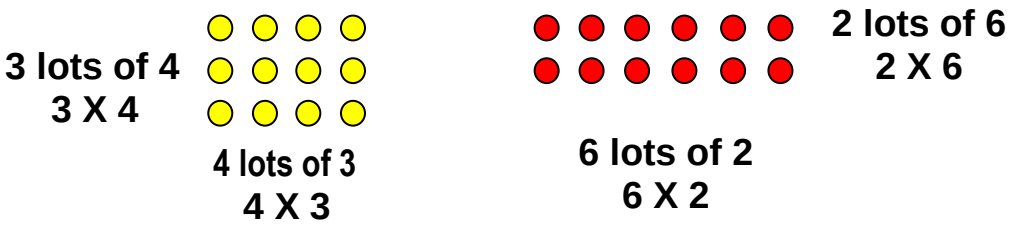
Stage 1: Step 1: Using number lines (Year 1 & 2):

This model illustrates how multiplication relates to repeated addition:

6 X 5 =

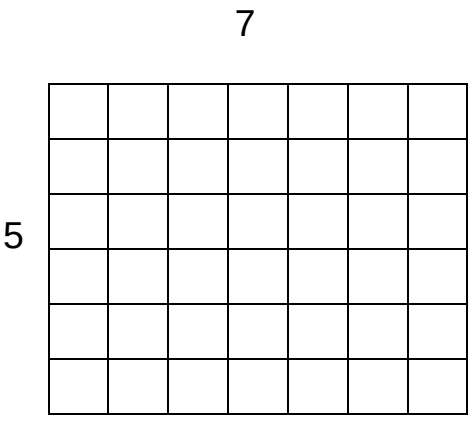


Step 2: Rectangular arrays (Year 1&2): If children are to be able to multiply successfully, they need to visualise multiplication as a rectangular array. In these examples, the rectangular arrays help children to understand why 3x4 = 4x3 and why 2x6 = 6x2**:**

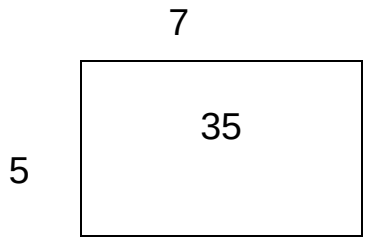


Rectangular arrays (Year 3/4): The area can be found by repeated addition, in this case 7+7+7+7+7. Children should then commit 7 x 5 to memory and know that it is the same as 5 x 7:

7 x 5 = 35



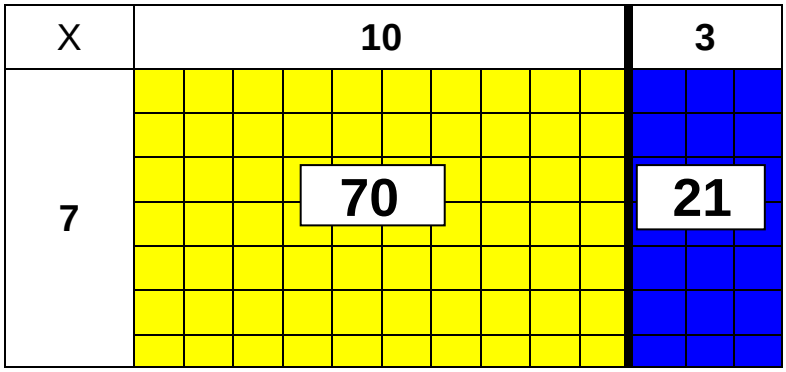
Area models (Year 3/4): Focus on multiplication facts, rather than use of repeated addition. For example:
7 x 5 = 35



Stage 2: Mental multiplication using arrays and partitioning to multiply two-digit numbers by a one-digit number (Year3/4):

Example 1 uses an **array,** to show that 13 x 7 is the same as (10x7) + (3 x 7):

13 X 7



NB: the squares are used to ensure the children have a secure mental image.

Example 2 moves to use of a **'blank rectangle'** to illustrate that 13 x 7 = (10 x 7) + (3 x 7):

	10	**3**	
7	**70**	**21**	**= 91**

Example 3a shows progression to the **grid method** to multiply two-digit by one-digit numbers (TU x U), initially by partitioning into 10s and estimating the answer:

38 x 7 Estimate: 40 x 7 = 280

	10	10	10	8
7	70	70	70	56

Example 3b: now multiples of 10 are multiplied, using the **grid method** i.e. 38 x 7 is calculated as 30 x 7 (using knowledge that 3 x 7 is 21, so 30 x 7 is 210) plus 8 x 7:

38 x 7 Estimate: 40 x 7 = 280

	30	8	
7	210	56	= 266

Stage 3: Using the grid method to find the product of two-digit by two-digit numbers (TU x TU), a three-digit by two-digit number (HTU x TU) and them using decimal numbers, with children estimating their answers first (Year 5/6):

Example 1: multiplication of a two-digit by two-digit number (TU x TU):

38 x 14 Estimate: 40 x 15 = 600

x	30	8	
10	300	80	380
4	120	32	152
		=	532

The products of each row are added and then the sums at the end of each row added, to find the total product.

Example 2: multiplication of a three-digit number by a two-digit (HTU x TU):

138 x 24 Estimate: 140 x 25 = 3500

x	100	30	8	
20	2000	600	160	2760
4	400	120	32	552
			=	3312

Division

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally e.g. find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

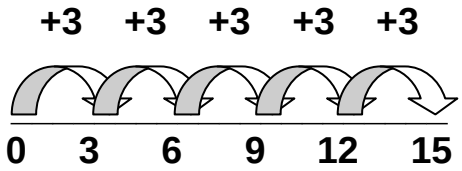
To carry out written methods of division successfully, children also need to be able to:

- understand division as repeated subtraction (grouping):
- estimate how many times one number divides into another e.g. how many sixes there are in 47, or how many 23s there are in 92;
- Know subtraction facts to 20 and to use this knowledge to subtract multiples of 10 e.g.120 - 80, 320 – 90

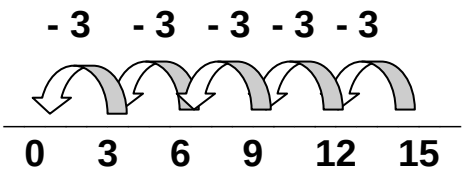
Stage 1: Number Lines (Year 1/2):

Example 1: counting on in equal steps based on adding multiples up to the number to be divided. E.g.:

15 ÷ 3 =



Example 2: counting back in equal steps based on subtracting multiples from the number to be divided.

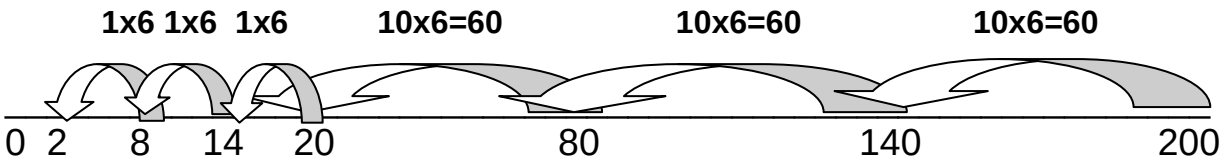


Stage 2: Counting back by chunking and using multiples of the divisor (Year 3/4):

This method is based on subtracting 'chunks' or multiples of the divisor. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. Chunking reminds children of the link between division and repeated subtraction.

Example 1 – subtracting using several 'chunks':

200 ÷ 6

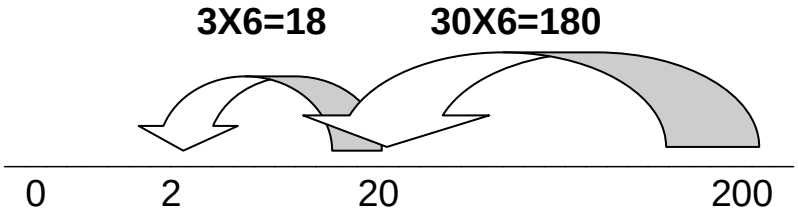


Answer 33 remainder 2

Example 2 – subtracting using the biggest multiples of the divisor

Children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples

200 ÷ 6



Answer 33 remainder 2

Stage 3: 'Expanded' method recorded in columns:

This method is based on subtracting multiples of the divisor, or 'chunks', from the number to be divided, the dividend. Initially children subtract several chunks, but with practise, they should look for the biggest multiples of the divisor they can find to subtract. Children need to recognise that 'chunking' is inefficient if too many subtractions have to be carried out – they should be encouraged to reduce the number of steps as shown in Stage 2, Example 2. The key to the efficiency of chunking lies in the estimate that is made before the chunking starts.

Example 1: 'Expanded' method for TU ÷ U (Year 3/4)

96 ÷ 6 =

96
60 10 x 6
36
36 6 x 6
0
Answer 16

Estimate:

- To find 96 ÷ 6, start by multiplying 6 by 10, to find that 6 × 10 = 60 and 6 × 20 = 120.
- The multiples of 60 and 120 trap the number 96. This tells us that the answer to 196 ÷ 6 is between 60 and 120.
- Start the division by first subtracting 60 (6 x 10) leaving 36, and then subtracting the largest possible multiple of 6, which is 36 (6 x 6), thus leaving no remainder.

Example 2: 'Expanded' method for HTU ÷ U (Year 3/4)

Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend. Estimating helps to choose a starting point for the division and to check the answer after the calculation

196 ÷ 6 =

196
60 10X6
136
60 10X6
76
60 10X6
16
12 2X6
4
Answer 32 r 4

Estimate:

- Before commencing this division children should estimate the answer by multiplying 6 by 10, 20, 30, to find that 6 × 30 = 180 and 6 × 40 = 240.
- The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40.

Example 3: Efficient 'expanded' method for HTU ÷ U (Year 3/4)

Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to this method, using the largest possible multiple of 10 as the initial multiplier.

	196	
-	180	30X6
	16	
-	12	2X6
	4	
Answer 32 R 4		

- Start the division by first subtracting 180 (30x6), leaving 16
- then subtract the largest possible multiple of 6, which is 12, thus leaving 4

Stage 4: Long division (Year 5/6)

The next step is to tackle HTU ÷ TU, which, for most children, will be in Year 6.

Example: How many packs of 24 can we make from 560 biscuits?

24	)560	
-	480	20 x 24
	80	
	72	3 x 24
	8	
Answer: 23 r 8		

- Start by multiplying 24 by multiples of 10 to get an estimate: 24 × 20 = 480 and 24 × 30 = 720; we know the answer lies between 20 and 30 packs.
- Subtract 480 from 560
- then subtract the largest possible multiple of 24, which is 72, thus leaving a remainder of 8

Useful Vocabulary

Bridge through a multiple: To go from one tens number to the next for example to move from the teens into the twenties or from the twenties into the thirties.

Complements: involves the image of an empty number line and then counting on from the smallest to the largest number, in steps, along the number line

Decomposition: adjust numbers to borrow from the next column so that a number can be taken from another.

Digit: each part of a number e.g. 23 is made up with the digits 2 and 3 but the value of the digit 2 is 20.

Divisor The amount by which a number is being divided.

Dividend The number being divided.

Factor: a number that divides another number exactly. For example all the factors of 24 are 1,2,3,4,6,8,12 and 24 because they all divide 24 exactly

Integer any positive or negative whole number

Inverse: the opposite. Addition is the inverse of subtraction and multiplication is the inverse of division and vice versa.

Multiples: numbers in a times table e.g. multiples of 10 are numbers that are in the ten times table e.g. 10, 20, 30.

Number bonds: pairs of numbers which total another. For example the number bonds for 10 are: 10 and 0, 9 and 1, 8 and 2, 7 and 3, 6 and 4, 5 and 5.

Operation: a process of doing something to numbers. The four operations of number are addition, subtraction, multiplication and division.

Partition: to split a number up so that each digit is recognised. For example 46 can be partitioned into 40 and 6

Product : The product of two numbers is the answer when they are multiplied together. For example the product of 7 and 5 is 35

Remainder: The number 'left over' after dividing numbers that do not divide exactly. For example 7 ÷ 3 is 2 remainder 1 (2 r 1)